LECTURE

ON

Gravitation and Centrifugal Forces,

BY

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GRAVITATING FORCE. — *Its Two Manifestations—Falling Bodies—Universal Tendency of Worlds to fall to each other—Three Laws of Gravitating Force — When Discovered.*

CENTRIFUGAL FORCE.—*Antagonistic to Gravity—Universal Tendency of Worlds to Move in Straight Lines, instead of Curved Orbits—Curvilinear Orbits the Result of Two Forces—Four Orbits possible, viz., the Circle, Ellipse, Parabola and Hyperbola — Under what Circumstances each is produced—Measure of Projectile Forces—Illustrated by Whirling Weights—Revolving Worlds—Law of Distance and Velocity—Weight of Worlds—Celestial Mechanism an Exhibition of Infinite Wisdom and Power.*

1.—GRAVITATION is a force exerted between all particles of matter in the universe. Two or more particles or masses of matter, placed in contact, have a mutual tendency to press towards each other's centers. This pressure, however great may be the difference in the quantities of matter, is equal and in opposite directions; for instance, one pound weight exerts the same pressure, upon the surface of the earth, as the earth exerts upon the one pound weight; and as the two forces press in directions opposite to each other, the two bodies must remain at rest.

If one kind of matter exerted a greater force than another in their mutual pressures, then the bodies, compounded of these different kinds, would necessarily move with a uniformly accelerated motion in the direction of the greater pressure. This may be represented by two steam vessels of equal weight in contact, pressing in opposite directions: if the force of steam on the two boats is equal, they will remain at rest; if the force of one preponderates over the other, there will be, not only a pressure, but a motion in the direction the greater force is exerted.

2.—Particles and masses of matter, not only exert the force of pressure when in contact, but when separated they manifest a mutual gravitation towards each other. By virtue of this force, when not restrained, they will move towards each other with a uniformly accelerated motion. If a mass of matter, elevated above the surface of the earth, is at liberty to move, it will fall towards the surface, while the earth, at the same time, will rise to meet it; but the velocities of the two moving bodies will be inversely proportional to their masses; that is, if the masses of the two moving bodies are equal, their velocities towards each other will be equal; if the mass of one body is twice that of the other, its velocity will be one-half; if its mass is three times greater, its velocity will be one-third; if its mass is one-hundred times greater, its velocity will be one-hundredth part of that of the smaller, etc.

The quantity of motion in a moving body is equal to the velocity multiplied into the mass; hence, when two bodies move towards each other by virtue of their mutual gravitation, their quantities of motion are always exactly equal. Therefore, when they meet, they will mutually destroy each other's motions, and remain at rest.

Thus it will be perceived that the force of gravitation is exhibited in two ways, namely, by pressure and by motion. Both of these effects are used to measure the force. When gravity is measured by pressure, the result is called weight, and is represented by the number of ounces, pounds, tons, etc. When it is measured by motion, the result is determined by the space through which a body moves by gravitation in a given time. Newton demonstrated, by careful experiments, that every kind of matter near the surface of the earth, falls from a state of rest, through equal spaces in equal times, when resistance of the atmosphere and other disturbances are removed.

3.—The space through which one body falls from a state of rest towards another, in one second of time, is one of the astronomical measures of gravitation. The space is usually estimated in inches or feet. The space through which a body falls from rest in this latitude, is sixteen feet and one inch, or, 193 inches in one second.

4.—In adopting this measure of the force of gravity, there are three laws which should be well understood and carefully observed.

FIRST LAW.—*The space through which one body, by virtue of its gravitation, moves towards another in one second, does not depend upon the mass of the former, but is the same whatever may be its mass, if the distances are the same.*

Thus, the Sun gravitates towards Jupiter, and the earth gravitates towards Jupiter also; but though the mass of the Sun is 314,760 times greater than the mass of the earth, yet the gravitation of the former towards Jupiter is exactly equal to the gravitation of the latter towards him, when the Sun and earth are equally distant from him; in other words, the Sun and earth, when at equal distances from Jupiter, move through the same number of inches or parts of inches towards Jupiter in one second of time. And such must always be the case, whatever may be the differences of the masses of any number of moving bodies towards another body.

SECOND LAW.—*The space through which bodies, by their gravitation, move towards another in one second, is proportional to the mass of the latter, if the distances between the former and latter remain the same.*

Thus, if the mass towards which a body gravitates is doubled, quadrupled, &c., the number of inches which the gravitating body will pass over in one second, will be doubled, quadrupled, &c. Suppose that the Sun and Jupiter are at equal distances from Saturn; as the Sun contains 1,018 times more matter than Jupiter, then whatever may be the number of inches through which Saturn may gravitate towards Jupiter, in one second of time, it will gravitate towards the Sun, in the same time, 1,018 times that number of inches.

THIRD LAW.—*If several bodies, at different distances, gravitate towards one body, the spaces passed over by the former, in one second of time, will be inversely proportional to the square of their distances from the latter.*

Thus, if the distance of the surface of the earth from its centre be called 1, and the space passed over by a falling body near the surface, is 16 feet, in a second of time, then at twice such distance from the centre of the earth, a body would fall one-fourth of 16 feet in one second. At three times such distance, a body would fall one-ninth of 16 feet. At four times the distance, one sixteenth of 16 feet, or one foot. At sixty times the distance, or sixty radii of the earth, (which is a little more than the moon's distance,) a body would fall, in one second, the one-thirty-six hundreth part of 16 feet, or about one-twentieth part of an inch. The Sun, being, in round numbers, nearly 400 times more distant from the earth than the moon, falls towards the earth, the one-hundred and sixty-thousandth part of the one-twentieth of an inch, which is equal to the thirty-two-hundred-thousandth part of an inch, in one second of time. While Neptune, being thirty times more distant from the earth than the Sun, falls towards the earth, in one second, 900 times less than that of the Sun, or the two-thousand eight-hundred and eighty-millionth part of an inch.

5. Perhaps this audience may inquire, how this law of gravity is known to be true? How is it known that the force called gravitation decreases as the square of the distance increases? Who has ascended the heights of heaven to experiment upon this force? Who, at the vast distances of the moon, sun and planets, has observed any phenomena by which he could exactly determine the fractions of an inch through which bodies would fall towards the earth in one second of time? The answer is that all mankind, *of every age,* have beheld the phenomena which include the data necessary for the solution of this grand problem. All have beheld the moon circling itself around the earth, at the distance, in round numbers, of about 240,000 miles, or at the distance from the earth's centre, of 60 semi-diameters of the earth. All might have known that a body must move in a straight line, when once put in motion, unless deflected from that straight line by some force. All might have known that the orbit or path of the moon, was curved towards the earth, being deflected every moment from a straight line, called its tangent, into a curved line. All might have known, by a simple calculation, the exact amount of this deflection from the tangent to the curve in one second of time. Yet, strange as it may appear, this simple phenomenon, though observed for thousands of years, was not comprehended till the immortal Newton, some two centuries ago, solved the simple problem. We say, simple problem, because it is now so easily solved; but in the days of Newton, it was considered the greatest scientific discovery that ever was unfolded to the mind of man.

Thus we see that the same force which makes a stone fall 193 inches in one second, causes the moon to fall 3,600 times less, during the same time. If gravitation did not vary in its intensity with the distance, then the moon would fall with the same velocity as a stone near the earth's surface; if the velocities decrease merely as the distance, then the moon, being 60 semi-diameters of the earth distant, would fall 60 times slower: but its falling velocity is known to be 60 times 60, or 3,600 times slower, being, as before stated, only about the one-twentieth of an inch in a second.

The mean semi-diameter of the earth is about 3,956.2 miles: the mean distance of the Sun, according to the new value is about 91,430,000 miles. The latter distance, divided by the former, gives a quotient of 23,110.6; that is, the Sun's distance is equal to 23,110.6 semi-diameters of the earth; this squared or multiplied into itself is equal to 534,099,832. If the Sun contained only the same amount of matter as the earth, then the latter would be deflected from the tangent of its orbit into a curve towards the Sun with a velocity 534,099,832 times less per second than that of a falling body near the earth's surface; but observation proves that the earth falls towards the Sun with a velocity several hundred thousand times greater than it would on the above supposition, its deviation from the tangent of its orbit being about the one-eighth of an inch per second; or more accurately, the earth falls towards the Sun about 1,697 times slower than a stone falls near the surface of the earth. Now if 534,099,832 be divided by 1,697, the quotient will be about 314,760; that is, the Sun contains 314,760 times more matter than the earth; *and thus the earth falls towards the Sun 314,760 times* faster than it would if the mass of the Sun were only equal to that of the earth.

Neptune being about 30 times the distance of the earth from the Sun, falls towards the latter, with a velocity about *900 times less than the falling velocity of the earth towards that luminary.*

A planet, twice the distance of the earth from the Sun, will fall towards him four times less distance in one second; at three times the distance, its falling velocity will be nine times less; at four times the distance, sixteen times less; at ten times the distance, one hundred times less; &c., &c.

Centrifugal Forces.

6th. As we have examined the law of gravitation, and determined a proper measure of its force for any given quantity of matter, and for any given distance, next, let us inquire into the nature of those opposing forces by which the planets and satellites are prevented from falling to one common centre.

7.—Every mathematician is well acquainted with the law of the composition of forces and motions. If a body is acted upon by two forces, it will generally have a resultant motion intermediate in direction, or between the directions which would have resulted from the action of either force singly. For instance, if a ball be projected on a smooth plane of ice, its path would be a straight line; but if the same ball be blown by a strong wind, at right angles to its projection or path, it would deviate from a straight line into a curve; or, if the ball be attached to a cord 100 feet long, and the other extremity be fastened to a pivot, 100 feet from the straight line which the ball is describing by virtue of its projection, the ball will immediately deviate from such straight line, or its former path, and will describe a circle around the pivot as its centre of motion. In this case, there are two forces which produce the circular motion; one is the projectile or propelling force; the other is the central or pulling force of the string; the first imparts the impulse or progression, and then ceases to act; the second acts continually and uniformly from a central point, as steam or gravitation. The space through which the body would travel in a straight line, in one second, is a proper measure of the projectile force: the perpendicular distance from the end of such straight line reaching the curve, at the end of one second, is the true measure of the fall towards the centre.

8.—All the planets and satellites of our system are under the influence of a projectile force, or a force which originally gave them motion, and then ceased to act; the same as powder ceases to act upon a ball, after the moment of explosion. Without a resisting medium, or some other extraneous cause, the planetary bodies would have continued to move for ever, with a uniform velocity, in the straight line in which they were projected. But gravitation every moment deflects them from this right line into curvilinear orbits. The nature of these orbits will depend upon the intensity and direction of the two forces.

9.—1st. If the motion arising from the projection is at right angles to the motion arising from gravitation, and if the former is exactly proportioned to that of the latter, so as to keep the body at all times at the same distance from the centre, its orbit will be a circle.

2nd. Though the projectile velocity is exactly such as would produce a circular orbit, when operating at right angles to the central force, yet, if operating at an acute or obtuse angle to the latter force, the body must describe an ellipse; and in this case, its mean velocity will be equal to that of its projection, and the point in the ellipse will be the mean distance of the body from the central force.

3rd. If a body is projected at right angles to the central force with a velocity insufficient to produce a circular orbit, it will describe an ellipse of which the point of projection will be the aphelion.

4th. If a body is projected at right angles to the central force, with a velocity somewhat too great for the central force to deflect it into a circle, it will revolve in an ellipse of which the point of projection will be the perihelion.

5th. If a body is projected in a line making an acute or obtuse angle with the direction of the central force, and with a velocity somewhat smaller or greater than that which, (if at the same distance, and acting at right angles,) would produce a circle, it will describe an ellipse.

6th. If the projectile velocity is equal to that which the body would acquire in falling from the point of projection to the sun, it will describe a curve called a parabola; the body, in this case, would follow one of the branches of this curve with a retarded velocity far beyond the limits of our solar system, never to return to it again.

7th. If the projectile velocity is greater than that which a body would acquire in falling from the point of projection to the sun, it would describe a curve called a hyperbola. As the branch of this curve in which the body would move, extends, like the parabola, to an infinite distance, the body never could return again to the solar system.

10. The circle, ellipse, parabola, and hyperbola, are the only possible orbits or curves, in which bodies can move, under the influence of projectile and gravitating forces.

At a given distance, there is only one velocity of projection, which will produce a circular orbit; and there is only one velocity which will cause a body to describe a parabola: but there is an infinite number of velocities which will produce an infinite number of elliptic orbits between a straight line and a circle. And there is also an infinite number of velocities which will cause bodies to describe an infinite number of hyperbolic curves of different forms.

All the planets and satellites of our system describe ellipses which do not vary much from circles. Some of the comets describe ellipses very eccentric; and some of them are supposed to describe parabolas and hyperbolas.

11. Let us next determine a measure for projectile forces, that they may be properly adjusted to any given central forces.

For the sake of simplifying our statements, let us suppose bodies to move in circles around central forces. It is required to determine what effect different velocities and distances will have in relation to central forces.

12. Let a string one foot long be attached to a ball of lead of the weight of one pound; let it be whirled around in a horizontal plane, and it will have a tendency to recede from the centre: the greater the velocity of revolution, the greater will be the force pulling upon the string in a direction outwards from the centre. This is called the centrifugal force. When the ball performs a revolution in 1.111379965 seconds, *it will become stretched,* by the circular velocity, equal to the force of its weight; that is, the centrifugal force will be equal to one pound. Let the velocity of revolution, which generates one pound of centrifugal force, be represented by 1.

Suppose the string, connecting the ball with the centre, to be varied in its length, while its velocity remains the same, what will be the law of force exerted upon the string? Both experiment and calculation demonstrate that,

With the same velocity, the centrifugal force varies inversely as the length of the string.

Examples: A string, one foot long, with a certain velocity, generates a centrifugal force of one pound. A string, two feet long, the velocity remaining unchanged, gives a force of one-half of a pound; three feet long, one third of a pound; ten feet long, one-tenth of a pound, &c., &c.

The philosophy of this law will be seen, by reflecting that a body, revolving with the same velocity, *at twice the distance* from the centre, is, when the arcs are small, *deflected from* the tangent to the curve, through a space only one-half as great, in the same time, as that through which the nearer body to the centre is deflected. At three times the distance the deflection is only one-third of that of the nearer body. At ten times the distance the deflection is only one-tenth of that of the nearer body, &c., &c. The forces exerted upon the string being as the deflections; and the deflections being inversely as the distances: therefore the centrifugal forces must be inversely as the distances.

Next, suppose that the velocity of the revolving ball is varied, while the length of the string, connecting it with the centre, remains the same. What will be the law of force exerted upon the string? In this case, as in the former, both experiment and mathematical calculation prove that,

When the length of the string remains the same, the centrifugal force varies directly as the square of the velocity.

Examples: A string one foot long, with a velocity equal to one, generates a centrifugal force equal to one pound: if the same ball, at the end of the string, is made to revolve with twice the velocity, the force exerted upon the string will be four pounds: if the velocity is three times as great, the string will be stretched nine pounds: if the revolving velocity is five times as great, the tension on the string will be twenty-five pounds: if the circular velocity of the ball is 600 times as great, the force exerted upon the string will be 360,000 pounds.

The reason of this law is apparent, by reflecting that in small arcs a body, revolving in a circle with a velocity equal to one, will be deflected, in a given time, *from the* tangent of its orbit to the curve, *through* a space which may be *represented* by one. With twice the revolving *velocity,* the deflection, in the same *time,* will be four times as far: with three *times* the revolving velocity, the deflection *from* the tangent, in the same time, *will* be nine times as far: etc., etc. The deflections being as the square of *the velocity,* therefore the central forces, *which* produce these deflections, *must* be *as* the square of the velocities: but in circles the central and centrifugal *forces,* balance each other and are equal; therefore, the centrifugal forces must be as the square of the velocities.

A body, having twice the revolving velocity, will, during the first half of the given time, be deflected through a space towards the centre, equal to that which it would have passed over if the velocity had not been increased; during the last half of the given time, it will be deflected three times as far as during the first half of the time; hence the whole deflection from the tangent will be equal to four.

If the circular velocity is three times as great, the deflection from the tangent must be nine times as far in the same time; for, during the first third of the time, the deflection will be one part of the space equal to one-ninth of the whole; the next third of the time, the body will be deflected through three parts more; and the last third of the time, it will be deflected through five parts more; one, three, and five, being added together, will be equal to nine. These are the philosophical reasons why the deflections and forces vary directly as the square of the velocities.

Falling bodies observe this same law: the first quarter of a second a stone falls 1 foot; the next quarter of a second it falls 3 feet; the third quarter it falls 5 feet; the fourth quarter 7 feet: one, three, five, and seven, added together, make 16 feet, hence, bodies fall 16 feet in one second.

13.—By a combination of the law of distance with the law of velocity, a law still more general may be obtained, applicable to any given length of the strings, and to any given velocity of the whirling ball. This law may be expressed as follows:—

The tensions of strings vary directly as the squares of the velocities of the revolving weight, attached to their ends, and inversely as their lengths.

Examples:—1. A string, one foot long, with a one pound weight attached, and having a velocity of one, or which performs one complete revolution in a horizontal plane in 1.111379965 seconds, will exert a force of tension on the string equal to 1 pound.

2. A string, 4 feet long, with a 1 pound weight attached, and whirling horizontally with a velocity 6 times greater than that in the first example, will exert a tension or centrifugal force on the string equal to 9 pounds; for if we square the velocity, *or multiply it into itself,* and divide the product by the length, the quotient will be 9; thus,

$$\frac{6\times6}{4}=9$$

3. If the weight is attached to a cord 10 feet long, and whirled with a velocity 20 times greater than that in the first example, it will generate a centrifugal force outwards from the centre of motion, equal to 40 pounds weight; that is, the horizontal strain upon the cord will be equal to a 40-pound weight, having no motion. This is determined in the same manner as the preceding; thus, $\frac{20\times20}{10}=40$

4. A string, 400 feet long, attached to a 1 pound weight and having a horizontal revolving velocity 30 times greater than that in the first example will strain the cord equal to $2\frac{1}{4}$ pounds;

$$\text{thus, }\frac{30\times30}{400}=2\tfrac{1}{4}$$

To determine the utmost tension that can be imparted to a string of given strength—

When the strength of strings is made to vary inversely as the squares of their lengths, then the greatest revolving velocities which they will admit of, will vary as the inverse square roots of their lengths.

EXAMPLES.—1. A pound weight attached to a string of a certain given strength one foot long, and revolving in a horizontal plane, with a velocity represented by one, will support, at its greatest possible tension, before breaking a centrifugal force represented by one.

2. A string four times as long will, according to the assumed hypothesis be sixteen times weaker; (one-sixteenth being the inverse square of its length;) and the greatest revolving velocity which it can endure, without breaking, will be one-half the velocity given in the first example, (one-half being the inverse square root of its length.) The string, representing the central force, being sixteen times weaker, the centrifugal force, necessary to balance it, must also be sixteen times weaker; but the length 4, and the velocity $\frac{1}{2}$, will generate a centrifugal force equal to one sixteenth; or

$$\frac{\frac{1}{2}\times\frac{1}{2}}{4}=\frac{1}{16}$$

3. If the string is 400 times longer than that in the first example, the assumed hypothesis will make it 160,000 times weaker; ($\frac{1}{160,000}$ being the inverse square of its length;) and the greatest circular velocity, which it is capable of enduring, without breaking, will be equal to one-twentieth of that given in the first example (one-twentieth being the inverse square root of its length). In this case, the greatest possible tension which the string will endure without breaking, is the one hundred and sixty-thousandth part of that in the first example; but the length 400, and the velocity one-twentieth, will exactly generate this force of tension; or,

$$\frac{\frac{1}{20}\times\frac{1}{20}}{400}=\frac{1}{160,000}\text{th}$$

These simple mechanical laws are applicable to the revolving wheels of machinery. If the strength of the spokes, or of the other materials of wheels, and their diameters, are known, the exact revolving velocities, which they are capable of enduring, can, by these laws, be calculated.

But the most interesting exhibition of these laws will be seen in their application to the celestial machinery of the universe.

If a one-pound weight or ball be attached to a string, reaching from the centre of the earth to its equatorial surface, its length would be 3962.81 miles, which would be equal to 20,923,637 feet. If this weight be whirled around the centre of the earth from west to east, so as to complete one revolution in 84 minutes and 43.7146 seconds, it will generate a centrifugal force, upwards from the centre, equal to one pound, which being exactly equal to its weight, the ball would be exactly balanced between the two forces, and would have no tendency to fall, but would continue to circulate around the earth, as long as its velocity continued the same. In other words, if the earth were made to rotate on its axis once in 84 minutes 43.7146 seconds, bodies at the equator would lose all weight.

I have already stated, that a ball of one